The construct of anchoring – an idea for ‘measuring’ interdisciplinarity in teaching[1]

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Abstract

The paper discusses a theoretical and methodological construct – the construct of anchoring – which is originally developed in the context of using history of mathematics and/or philosophy of mathematics in mathematics education. The idea of the original use is to see if students’ historical and/or philosophical discussions are somehow rooted in or based on their mathematical content knowledge regarding the actual mathematics in designed teaching modules on specific cases from the history and philosophy of mathematics. The construct builds on (i) Anna Sfard’s theory of commognition (a contraction of communication and cognition) which in itself is a discursive approach to learning and on (ii) methodological triangulation between various gathered data sources. Based on a description of the original use of the construct in an empirical study carried out in Danish upper secondary school, I aim at arguing for the use of this construct as a way of ‘measuring’ the level of interdisciplinarity in cross-curricular/interdisciplinary teaching activities at (at least) secondary and tertiary educational levels and between practically any combination of two or more disciplines or subjects. The ‘measuring’ of the level of interdisciplinarity present in the implementation of teaching activities will be based on Eric Jantsch’ taxonomy of interdisciplinarity.

Introduction

The first use of the word *interdisciplinarity* is believed to stem from 1937 (Klein, 1996) and up through the 20th century the topic of interdisciplinarity is one that attracted a great deal of attention, within the natural and social sciences as well as the Humanities, and this for very good reasons. Often it is so today that when truly breaking and edge-cutting research takes place, it is in the intersections or boarder regions of the already existing and well-established subject areas. The clear-cutting argument for interdisciplinarity is of course that the problems of our society and the real world in general do not respect and confine to the disciplinary boundaries set up by us humans

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[1] This is an extended version of a paper presented at the 11th International IHPST and 6th Greek History, Philosophy, and Science Teaching Joint Conference in Thessaloniki, 2011.

(Mark E. Kann in Klein, 1990). The topic of interdisciplinarity has become a research field in itself from a meta-perspective point of view, discussing questions such as how interdisciplinary research activities are being carried out, what actually constitutes interdisciplinary research, the political dimension of the topic, etc. Of course, also the educational dimension of interdisciplinarity (including also cross-curricular teaching) is begin touched upon, but as of yet this dimension still has a ways to go regarding the development of didactical, theoretical, and methodological tools for evaluating the effect (and efficacy) of actual implementations of interdisciplinary teaching activities between two or more subjects.

A framework for discussing interdisciplinarity

When discussing interdisciplinarity, Jantsch' (1972) taxonomy of distinguishing between five different kinds of 'disciplinarity' is often used. At each end of the spectra we find (1) *multidisciplinarity* and (5) *transdisciplinarity*, respectively, the first resembling the complete participation of disciplines that we already find in the educational system and the other resembling interdisciplinarity to such a degree that the individual disciplines become invisible, e.g. when biology and chemistry become biochemistry, and so on. According to Ulrichsen (2001), the three middle ones are what may be considered 'true' types of interdisciplinarity in an educational setting: (2) *pluridisciplinarity*; (3) *crossdisciplinarity*; and (4) *interdisciplinarity* (*proper*). Pluridisciplinarity is when disciplines cooperate about a common theme, e.g. "the 1950s" where each discipline will treat this time period from only their own perspective. Crossdisciplinarity is when one or several disciplines act as *supporting* disciplines for another, e.g. when biology will rely on mathematical methods to model a purely biological problem and possibly also draw in some ethics from philosophy to clear the humanistic aspects. Interdisciplinarity proper, in Jantsch' terminology, is however when all involved disciplines are subject to a common outer principle or problem and focus moves from begin on the individual disciplines to being on the *connections* between them. The situation for these three types (2,3,4) is illustrated in figure 1.

Since Ulrichsen (2001) defines the three middle types of disciplinarity in Jantsch' model as the ones relevant for actual types of interdisciplinarity in teaching and education, I shall confine myself to consider only these three in the remaining part of the paper.

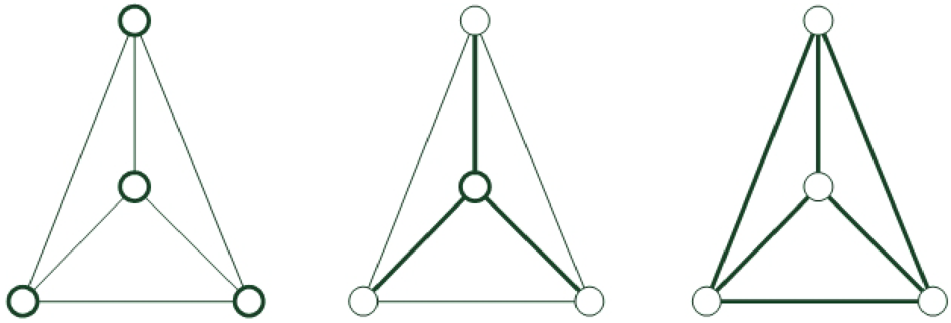

Figure 1. From left to right: Jantsch' pluridisciplinarity, crossdisciplinarity, and interdisciplinarity proper illustrated as the emphasis (by dark coloring) on subjects (circles) and connections between subjects (lines) (Jensen, 2010).

Sfard's discursive approach to learning

A basic assumption of Sfard (2008) is that understanding and learning is the occurrence of a form of change in one's thinking. According to Sfard, the best candidate for the precursor of thinking is *communication* (Sfard, 2008). That is to say, she defines thinking as "an individualized version of (interpersonal) communication" (Sfard, 2008, p. 81), thereby also referring to communication not having to be verbal or outspoken, but that it can take place inside the mind of an individual (e.g. in the process of sense-making) as well as between two or more individuals. The term *commognition* is introduced as a contraction of communication and cognition (commognition = communication + cognition), stressing the fact that these two processes are different intrapersonal and interpersonal manifestations of the same phenomenon. Her next step is to define a *discourse* as "the different types of communication, and thus of commognition, that draw some individuals together while excluding some others" (Sfard, 2008, p. 91). In the context of mathematics education, Sfard then roughly reasons that *mathematics is a way of thinking*, thinking is a form of communication, communication is a discourse, and therefore mathematics is itself a discourse. Thus, if thinking is what changes when one understands or learns, this means that understanding or learning mathematics is the same as changing the discourse (Sfard, 2008).

Due to the interdisciplinary (read: non subject specific) generality of the philosophical foundations of Sfard's theory – it builds on elements of behaviorism, cognitivism, acquisitionism, and participationism – the word 'mathematics' of the line above in italics may be changed with practically any other subject that can claim to have its own theories, methodologies, etc. Thus, Sfard's discursive approach is applicable to understanding and learning in general.

Defining the construct of anchoring and the notion of potential anchoring points

In previous studies on the integration of history of mathematics in mathematics education (Jankvist, 2010, 2011a; Jankvist & Kjeldsen, In Press) and philosophy of mathematics in mathematics education (Jankvist, 2011b), the construct of anchoring has played the role of validating an actual anchoring of students' discussions of meta-perspective issues (meta-issues) of mathematics (i.e. historical and/or philosophical/epistemological issues, but possibly also sociological, psychological, application-oriented, etc.) in the related inner issues (in-issues) of mathematics (i.e. the mathematical content knowledge as for example that referring to mathematical concepts, theorems, methods, algorithms, notions of proof, etc.).[2] Sfard's theory of commognition was applied in these studies to look at the students' discussions and reflections of the meta-issues. Also, the discursive approach of Sfard offered appropriate lenses for looking at gathered data, e.g. for video recordings of students' group work during sessions, since the discussions and reflections of the students could be viewed as following different discourses.

In this setting one may in general talk about two different – yet related – discourses being present: a *meta-issue discourse* and an *in-issue discourse*. *Anchoring* is then defined as something that solidifies and substantiates the treatment of meta-issues on a basis of knowledge and understanding of the related in-issues, for example, by revealing insights about the meta-issues that could not have been accessed or uncovered without knowing about the in-issues, or by providing in-issue evidence for meta-issue claims or viewpoints (Jankvist, 2011a). Building on Sfard (2008) it may be argued that *reflection,* like understanding, is also change in discourse. In this sense, a change or *shift in discourse* can be regarded as a necessary condition for the occurrence of anchoring in a discussion. If such a shift took place between a meta-issue discourse and an in-issue discourse it was regarded as a *potential anchoring point* (Jankvist, 2011a). Here *point* refers to a point in time of the discussion and the word *potential* refers to the stance that a shift in discourse is neither a guarantee nor a sufficient condition for anchoring. So, once having identified a potential anchoring point, further steps are needed in order to either *verify* or *reject* it as an actual instance (or episode) of anchoring. An appropriate way of dealing with such verification or rejection is by means of so-called methodological triangulation (Cohen & Manion, 1994), i.e. to consult various other data sources gathered during the actual implementation of the activity (e.g. students' tests, questionnaires, hand-in tasks and essays, etc.) – possibly also in combination with Sfard's notion of 'word use', for concrete examples, see Jankvist (2011a).

[2] For a general discussion of meta-issues and in-issues, see Jankvist (2009a; 2009b).

An actual example of anchoring from a use of history of mathematics

In a setting of working with the history of public-key cryptography and RSA in an upper secondary mathematics class (for more detailed descriptions, see Jankvist, 2009a; 2011a), student groups were to discuss and write essay answers to a the question of the mathematicians' "personal motivation for working with the parts of cryptography and/or number theory with which they did" and how "the personal motivation for each of them relate to the discussion of inner and outer driving forces." The terms *inner* and *outer driving forces* refer to what drives the development of mathematics (or science in general), and whether these forces may be considered to be intrinsic or extrinsic to the discipline (e.g. Jankvist & Kjeldsen, In Press). For example, unsolved questions, conjectures, etc. within mathematics may act as inner driving forces for the discipline, whereas the need for secure communication, fast transmission of data, and other societal needs make up external circumstances that may drive forth the development of mathematics from the outside.

In the brief excerpts from the transcribed student discussion that we shall see, two students, Gloria and Harry, are discussing the personal motivation of the American mathematician, Withfield Diffie, who developed the idea behind public-key cryptography (for a precise description of this see, Jankvist, 2011a). The discussion begins with a third student, Lola, paraphrasing a quote of Diffie given in the students' teaching material:

Lola: Check this out: "Diffie recalls the day in 1975 when he first got the idea for public-key cryptography like this: 'I walked downstairs to get a Coke, and almost forgot about the idea. I remembered that I'd been thinking about something interesting, but couldn't quite recall what it was. Then it came back to me in a real adrenaline rush of excitement. I was actually aware for the first time in my work on cryptography of having discovered something really valuable.'"[3] He is . . .

Harry: He is really interested in it himself, right?

Gloria: Yes, yes, but all of them are. There is actually an idea behind him producing this mathematics: it is for developing the Internet.

Harry: Yes, but nobody told him that he should do it to develop the Internet. It's like in wars, there are many . . .

Gloria: No, nobody . . . but I also think . . . . No, but the outer circumstances, it also has to do with the situation of the world and its state, and how far you are and what it can be used for.

Because the question has to do with personal motivation, Harry is trying to introduce a somewhat psychological meta-issue discourse into the discussion. To Gloria, however, the question of Diffie's

[3] Lola is reading aloud from the teaching materials that contain a quote that was originally taken from Singh (1999, p. 268).

personal motivation is tightly connected to the discussion of outer driving forces, because she sees Diffie as a visionary and as having 'higher' goals (e.g., with developing the Internet) than doing cryptography solely out of pure interest. Harry is not convinced, and the students go on to discuss other matters for a while. But eventually, Gloria comes back to her somewhat sociological meta-issue discourse of Diffie wanting to do this "for the sake of the world," as she phrases it elsewhere, now referring to the textbook explanation of outer driving forces:

Gloria: Try and listen to this: "Outer driving forces are understood as those forces which affect the research of mathematics from the outside."

Harry: They began this not knowing if it was going to lead somewhere or not. What he [Martin Hellman, a collaborator of Withfield Diffie] writes is that it's just idiots, who keep on trying, isn't that right? They continue because they are interested in it themselves.

Gloria: And because they want to succeed, so the world can evolve a little and the Internet can come to work.

Harry: But then you can say that about everybody, because everybody wants the world to evolve.

Gloria: That's not right. This guy who says: I want to play with prime numbers and I don't care if it can be used for anything.

Of course part of the discussion between Gloria and Harry concerns what is and what is not to be considered outer driving forces, but from the viewpoint of anchoring something interesting takes place in Gloria's final comment. From having repeatedly argued her point of view within her sociological meta-issue discourse, she now shifts to one involving mathematical in-issues (prime numbers) in order to substantiate her meta-issue claims. The 'guy' she refers to is the English mathematician and number theoretician G. H. Hardy and his viewpoints regarding number theory in relation to pure and applied mathematics, as expressed in his *Apology* from 1940 (Hardy, 1992), which the students had read about 2/3 of as part of the historical teaching module. By making this shift from a meta-issue discourse to a mathematical in-issue discourse, Gloria is eventually able to influence Harry to modify his view some.

Gloria's reference to prime numbers, simple as it may appear at first sight, constitutes a potential anchoring point – an instance in the discussion where meta-issue claims are substantiated and underpinned by references to mathematical in-issues. However, in order to verify (or reject) that this is an *actual* anchoring point, we need to refer to other data sources collected during the duration of the historical teaching module. In the original study (Jankvist, 2009a) potential anchoring points for specific students were analyzed by triangulation with questionnaire data, student interviews, video recordings of student group work, students presenting mathematical proofs on the blackboard during class, and individual mathematical tasks on topics related to public-key cryptography and

RSA, in particular tasks on number theoretic aspects. For Gloria the triangulation done to verify the potential anchoring point as an actual one was based on a combination of questionnaire answers, a blackboard proof and mathematical tasks relating in one way or another to the notion of prime numbers (for a detailed account, see Jankvist, 2009a).

Application of the construct of anchoring to interdisciplinary teaching activities

From the above example, we may imagine a presence of anchoring in activities not only between mathematics and its history, but between practically any two disciplines, subjects, or topics. I shall provide a couple of constructed examples.

The first example stems from a student exam project from a course in ‘Modeling and Interdisciplinarity’ (called Nat802) held at University of Southern Denmark, where pre-service upper secondary school teachers in science and mathematics were to design interdisciplinary teaching activities for future use (for a description of the course, see the paper by Jankvist, Nielsen & Michelsen (2011) in the IHPST-2011 proceedings). As an example of an interdisciplinary activity between physics and sports, one student group gave that of deciding which jumping style for high jump (in athletics) is the most beneficial (Rasmussen & Hansen, 2010). Three jumping styles were examined: the straddle technique, which is a sideways or diving approach with the stomach directed towards the pole; the Western roll, which is a scissors-jump with the back towards the pole; and finally the most commonly used, the Fosbury flop, a backwards jump with the back towards the pole. After having practiced each of these styles in sports class, the students were to estimate the amount of energy used with each of them. One way of doing this is by applying the physics concept of *center of mass*. Say a certain upper secondary school pupil is capable of this then she will soon realize why the Fosbury flop is the commonly used one. The reason is that when bending your back backwards, you move your body’s center of mass outside the body, this resulting in a high jump where the body goes over the pole and its center of mass passes slightly beneath the pole. Thus, the amount of energy used for such a jump is the same as if the body had stayed stiff during the jump (and thereby passed under the pole). If a pupil is able to change her discourse from sports to physics and follow this line of reasoning, then surely this will give rise to an anchoring of her knowledge of sports (high jump) in her physics content knowledge. Following Jantsch’ taxonomy, we here find ourselves in a situation of crossdisciplinarity (3), where physics in a certain sense act as support for the pupil’s understanding of sports.

As a second example we may imagine an interdisciplinary/cross-curricular activity between mathematics and physics on the topic of infinitesimal calculus, in particular the part of this concerning derivation. Now, if on the one hand students involved in such an activity are capable of substantiating their physical arguments by referring to the mathematical definition and interpretation of derivation then there is an anchoring of physics in mathematics. And if on the other hand the students try to explain the mathematical meaning of derivation by referring to physical phenomena such as the 1^{st} derivative being speed and the 2^{nd} derivative being acceleration, then there is an anchoring of mathematics in physics. In this situation, where we have anchoring of both mathematics in physics and of physics in mathematics, we have a situation that resembles Jantsch' interdisciplinarity proper (4).

Main idea of the paper (in a nutshell)

Thus, the main idea of this paper may be laid out very briefly as follows: If in a teaching activity involving two (or more) subjects anchoring does not occur, then the activity may be characterized as pluridisciplinary; if anchoring occurs of only one subject in another, then the activity may be characterized as crossdisciplinary; and if anchoring occurs of one subject in the other and vice versa, then the activity may be characterized as interdisciplinary proper, using Jantsch' taxonomy. The situation is illustrated in figure 2.

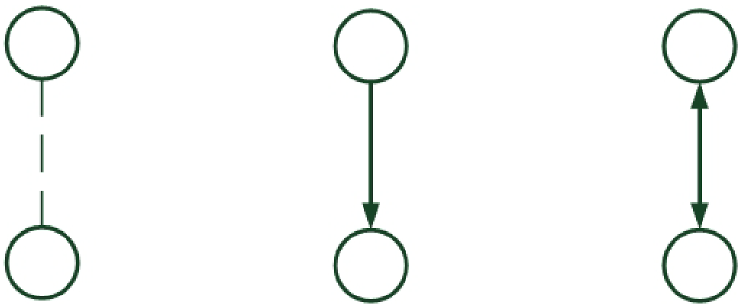

Figure 2. From left to right: Pluridisciplinarity, crossdisciplinarity, and interdisciplinarity proper between two subjects (circles) illustrated by the presence of anchoring (arrows), i.e. for the middle case the top subject is anchored in the bottom one, whereas for the right case both subjects are anchored in each other. (This model may of course be generalized to include more than just two subjects, but in terms of clarity it would become too complex if it (i.e. the mathematical complete graph) had to include all possible combinations of anchoring between the involved subjects.)

In actual teaching activities where anchoring occurs on several occasions, the number of verified anchoring points may of course be taken as a measure for the *level* of interdisciplinarity (proper) of the activity, if such a measure is wished for in order to, for example, compare one activity to another. The challenging aspect of 'measuring' the level of interdisciplinarity in an activity based on the construct of anchoring as described in this paper, is the verification or rejection of identified potential anchoring points as actual instances (or episodes) of anchoring – as also seen from the example with Gloria and prime numbers. However, once this tedious work of consulting one's set of data-sources is carried out, the classification of the achieved level of interdisciplinarity is easily identified, first within the three middle types of interdisciplinarity in Jantsch' taxonomy and next within interdisciplinarity proper itself, if needed.

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