

Communication competency as an indicator for mathematical giftedness

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Abstract

This paper addresses mathematically gifted students and their capacity to express themselves in regard to their mathematical problem tackling when in dialogue with a teacher. Through a case study of a student in 3rd grade—Celeste—it is argued that a well-developed communication competency may indeed be an indicator of mathematically giftedness. In doing so, attention is drawn to the problem of many teachers not being equipped for engaging in dialogue with such students and thus not being able to facilitate their mathematical learning in a productive and efficient manner. The analyses in the paper draw on the theoretical constructs of the Danish mathematics competencies framework (KOM) and the so-called Inquiry Co-operation Model (IC-Model).

Introduction

Indicators of mathematical giftedness are usually taken to be: unusual curiosity; ability to understand and apply ideas quickly; ability to see patterns; abstract thinking; being creative and persistent (Stepanek, 1999). In addition, Ernest (2011) mentions: mathematical self-confidence; self-efficacy; attitudes; and motivation. All of these indicators are connected by the student's ability to *communicate* his or her thinking to the teacher, or others, in a dialogue about mathematics. But despite the fact that communication is central to the above indicators, research rarely focuses on mathematical communication in relation to mathematical giftedness. In this paper we shall do so by presenting a case study of a 3rd grade student, *Celeste*. Besides being mathematically gifted, Celeste also had the ability to “talk mathematics” when dialoguing with adults.

Celeste was spotted as part of the Danish TMTM project (*Tidlig Matematikindsats Til Marginalgrupper*—Early Mathematics Intervention Program for Marginal Groups), an intervention study from 2015 and on, concerning as well low as high performers in 2nd grade school mathematics. In a period of twelve weeks, two low and two high performers took part in 48 individual lessons with a specially educated teacher. These teachers had a 30 hours course to learn how to use specially designed material (Lindenskov & Weng, 2013) as a background for starting dialogues with low and high performers about their belief and attitudes related to mathematics. Involved in the TMTM project were 82 such specially educated mathematics teachers and 281 students in the beginning of 2nd grade from 41 different schools in Denmark as well as a number of associated researchers. From the

observations that the researchers did in the TMTM project of how the teachers were acting in relation to the students during the interventions, we noticed Celeste. She was one of the very few high performers among the approximately 150 students in TMTM who were able—and motivated—to go into a learning dialogue with the teacher. Nevertheless, Celeste's own mathematics teacher, who may be characterized as an experienced primary school teacher, had neither identified Celeste as a mathematically gifted student nor as one of the high performers in her class. In the TMTM project this was by no means an isolated case; oftentimes local school teachers did not themselves flag the students that the embedded tests of the TMTM project identified as high performers.

During our interview-based interventions with Celeste, we have come to realize the importance of her mathematical communication competency in relation to her problem tackling competency (Niss & Højgaard, 2011). Time and again, we have witnessed that our attention to her way of communicating her thoughts concerning the mathematical problems presented to her have resulted in activating her other mathematical competencies, this leading to further mathematical understanding on her behalf. Clearly this only happens, if Celeste is communicated *with*. Hence, the underlying aim of this paper—besides pointing to students' communication competency as an indicator of giftedness—is to argue for the importance of mathematics teachers being observant to and actively facilitating gifted students' way of communicating their mathematical thinking.

On mathematical giftedness

Research studies on mathematically gifted students are much less in number than research on challenged students, i.e. students with mathematics-related learning difficulties. This is also stated by Szabo (2017), who has conducted a review of some 180 reports on research studies on mathematically gifted students from 1953 to 2014—this taking into account that no prevalent definition of mathematically giftedness exists (but we shall not go into this discussion in the present paper). Szabo, finds that the 180 research studies may be divided into five relatively different categories:

- gifted students' *performances* in mathematics,
- gifted students' *social situation* in school as well as *gender differences*,
- *teachers' perception* of gifted students in mathematics class,
- *definition and identification* of mathematically giftedness as well as national programs for mathematically gifted students,
- gifted students' *motivation* and *cognitive capacity*. (pp. 26-27)

As for the content of the research studies in relation to gifted student's performances, Szabo identifies four:

- gifted students in *heterogene* mathematics classes,
- *acceleration* of gifted students (through advanced courses, etc.),
- grouping gifted students according their *level*,
- gifted students' *attitudes* to different working methods. (p. 27)

Worth noticing is that In the description of these categories and areas there is indeed no focusing on dialogue and communication between teacher and student. This supports our

initial statements that focus on these two aspects of high performers in mathematics education have been somewhat neglected in the research literature—unlike the research literature on low performers, where several studies focussing on dialogue and communication do exist. Another aspect which is worth drawing attention to is that when it actually comes to dialogues with mathematically gifted students, the interlocutors are usually not student and teacher, but student and parent (e.g. Smutny, 2015).

Dialogue in learning mathematics

Already Pólya (1945/1990) was aware of the importance of dialogue in his book *How to Solve It*. In the preface to the first printing of his book, he indirectly focused on the importance of a dialogue through the teacher's ability to pose stimulating questions to the student:

But if he challenges the curiosity of his students by setting them problems proportionate to their knowledge and helps them to solve their problems with stimulating questions, he may give them a taste for, and some means of, independent thinking. (p. xxxi)

Pólya pointed to the importance of dialogues, and conversation in general, in the process of learning mathematics, and that the same applies when seeing a dialogue as an inquiry process:

Yes, mathematics has two faces; it is rigorous science of Euclid but it is also something else. Mathematics presented in the Euclidean way appears as systematic, deductive science; but mathematics in the making appears as an experimental, inductive science. (p. xxxiii)

Since Pólya's book on problem solving from 1945, there has been an increased focus on the importance of the conversation in the classroom, when teaching and learning mathematics as an inquiry process. Yet, the face of mathematics as a rigorous science learnt from solving mathematical items from a textbook is, as far as we have experienced, still dominant.

As the reader no doubt will know, Pólya offers four principles or steps for approaching a problem: understanding the problem; devising a plan; carrying out the plan; and looking back. Inspired by this, Skovsmose and Alrø (2002) proposed a framework of *Inquiry Co-operation Model*—referred to as the IC-Model. The IC-Model deals explicitly with the type of communication taking place between people working on (open-ended) mathematical tasks. Skovsmose and Alrø state:

Not any kind of communication can be characterised as a *dialogue*. In general terms, we describe a dialogue as an *inquiry process* which includes an exploration of participant perspectives as well as willingness to suspend one's preunderstandings—at least in a moment. (p. 15)

The characteristics of a dialogue as an inquiry process comes from the view of mathematics as *landscapes of investigation*. The IC-Model points out a cluster of eight elements that the teacher should be aware of when dialoguing with (gifted) students ready to go into an inquiry process in mathematical problem solving. These are:

1. *[G]etting in contact* involves inquiring questions, paying attention, tag questions, mutual confirmation, support and humour.
2. *Locating* has been specified with the clues of inquiring, wondering, widening and clarifying questions, zooming in, check-questions, examining possibilities and hypothetical questions.
3. *Identifying* involves questions of explanation and justification and crystallising mathematical ideas.
4. *Advocating* is crucial to the particular trying out of possible justifications, and it is closely related to arguing and considering.
5. *Thinking aloud* often occurs as hypothetical questions and expression of thoughts and feelings.
6. *Reformulating* can occur as paraphrasing, completing of utterances and staying in contact.
7. *Challenging* can be made through hypothetical questions, examining new possibilities, clarifying perspectives, and it can be a turning point of investigation.
8. *Evaluating* implies constructive feedback, support and critique. (ibid., p. 110, bulleting and numbers added by us)

Mathematical competencies and communication

The Danish competencies framework (KOM) defines a mathematical competency as (an individual's) "...well-informed readiness to act appropriately in situations involving a certain type of mathematical challenge" (Niss & Højgaard, 2011, p. 49). The framework consists of eight distinct, yet mutually related competencies, which can neither be possessed nor developed in isolation from one another. These are the competencies of mathematical: thinking; problem tackling; modelling competency; reasoning; representation; symbol and formalism; aids and tools; and not least communication. Each of the eight competencies has what might be thought of as a producing and an analyzing side to them. We briefly outline the competencies which we consider most important for the case study addressed in this paper.

The mathematical *problem tackling competency* involves the ability to detect, formulate, delimitate, and specify different kinds of mathematical problems, pure and applied both, as well as being able to solve mathematical problems in their already formulated form, whether posed by oneself or by others. The important thing to notice about this competency is that the word "problem" is relative to the person who is trying to solve it, what is a routine task for one person may be a problem for another and the other way around. When the problems addressed concern the extra-mathematical domain, the mathematical *modelling competency* often has to be activated. This consists of the ability to analyze the foundations and properties of existing models and to assess their range and validity. On the other hand, it involves being able to perform and utilize active modelling, including mathematization, in given

extra-mathematical contexts and situations. Embedded in both problem tackling and modelling is of course mathematical reasoning. The mathematical *reasoning competency* consists, first, of the ability to follow and assess mathematical reasoning, i.e. a chain of arguments put forward in support of a claim. It includes of course understanding what a mathematical proof is, the basic ideas of a proof, and when a chain of arguments does or does not constitute a proof. This also comprises being able to understand the role and logic of a counter example. Furthermore, the competency consists of the ability to actually devise, carry out and explain (valid) mathematical proofs and reasoning.

As the reader might well imagine, communication abilities are imperative to problem tackling, modelling and reasoning. The mathematical *communication competency*, firstly, consists of being able to study and interpret others' written, oral, or visual mathematical expressions or texts. Secondly, it consists of the capability to express oneself in different ways and at different theoretical or technical levels about mathematical matters, again either in written form, orally, or visually. Since written, oral, or visual communication in and with mathematics make use of various representations, the communication competency is closely connected to the *representing competency* – and also the *symbol and formalism competency*, since such communication often relies on mathematical symbols and terms. However, the communication competency goes further than the other competencies, since the communication happens between the sender and receiver, and their situations, backgrounds and prerequisites need to be taken into account for communication in the same way that purpose, message and media must.

Celeste's solution to a PISA task

In January, 2017, when Celeste was 9 years old and half way into 3rd grade, we presented her with a selection of challenging tasks during a session at the public school she goes to in the Copenhagen area. Celeste tells us that she is fond of mathematics, but that it is not something she has picked up at home. Her parents do not work with mathematics, she says. Her mother is a cosmetologist and her father an auto mechanic.

One of the first tasks we presented Celeste with is the following released PISA task. Among Danish 15-year olds (9th grade) in PISA-2012, 31.6% did *not* get this task correct (Lindenskov & Jankvist, 2013). The task as such is one that tests students' capability to reason proportionally. At the same time, it also tests simple arithmetics skills; which exactly depends on whether the student applies an additive or a multiplicative strategy.

Question: *You are making your own dressing for a salad. Here is a recipe for 100 millilitres (mL) of dressing. How many millilitres (mL) of salad oil do you need to make 150 mL of this dressing? Justify your answer. [PISA item PM924Q02]*

Salad oil:	60 mL
Vinegar:	30 mL
Soy sauce:	10 mL

Uffe: Should we try the one with the salad dressing? ... Have you tried baking a cake at home, and then you had to change the recipe? Maybe you wanted to double the size or something.

Celeste: I've tried that quite often.

Uffe: This is the same. We're just making a dressing. [*reads aloud the question*]

Celeste: Well, without... if you take out the others, so it was only the salad oil... or...

Uffe: Well, it's... [*points to the table*] Here is a recipe, you see that. It adds up to 100 mL, because all together these give 100. So, we want you to make a dressing of...

Celeste: 150 mL.

Uffe: So, how much do you have to put in of each [*ingredient*]? If it is to taste the same?

Celeste: Well...

Uffe: You can write over here, if you want. [*draws one more column of the table*] How do you figure this out?

Celeste: Maybe you could... [...]

Using the IC-Model, we see that Celeste pays attention to the problem formulation, she asks inquiring and clarifying questions (about the composition of the ingredients), which are the elements of “getting in contact” and “locating”.

Peter: Try to write 100 there. [*points to the original column in the table*] And then put 150 at the bottom of the one you are to do now. [*points to the new column*]

Celeste: So you should... if we only are to find out that it should give 50, then we have this plus this, right? [*points to the two columns*]

Uffe: Yeah, what do you mean?

Celeste: Well, if we have our 100 here. If we then find out somehow a way for it to give 50 instead of 100... then you could add it together with the other result you had... and then ...

Uffe: Okay, we can do it like that. So, you would prefer that we have 50 over here instead of the 150 [*points to the new column*] ... that is to find out how we make the 50. And how do we do that?

Celeste: Well, we could ... maybe we could minus ... or use the half of all the things. That would be it, if it has to add up to 100 in total.

Peter: That was pretty smart.

Uffe: Yes, it's a good idea. Try to write it down. You can write over there. [*points to the new column*]

Celeste: [*plots half of the original ingredients into the new column—cf. Fig. 1*]

Uffe: Should we make yet a column over here for the 150? [*draws yet a column*]

Celeste: Yes, okay! So, we say... So, this is 90, right. And this is 45 ... and then it gives 150.

Peter: That is super.

Uffe: Yes, this was very good indeed.

Spørgsmål 10

Du er ved at lave din egen dressing til en salat. Her er en opskrift på 100 milliliter (ml) dressing.

Salatolie	60 ml	30	90
Eddike	30 ml	15	45
Soyasauce	10 ml	5	15
	100	50	150

Hvor mange ml salatolie skal du bruge for at lave 150 ml af denne dressing? Begrund dit svar.

Figure 1. Celeste's solution to the PISA task.

In this next part of the dialogue we witness that Celeste considers different ways of tackling the problem, which is part of “advocating” and she does so while “thinking aloud”. Possibly Celeste could have chosen a more multiplicative proportional model (the next example supports this claim), but in this case she chooses a more additive approach, i.e. taking half the amount of the original ingredients and adding to the full amount—all aspects of the element of “identifying” in the IC-Model. Throughout the entire dialogue Celeste continuously stays in contact with us as interlocutors and she is open to constructive criticism and support in relation to her problem solving—“reformulating” and “evaluating”. From a mathematical competencies point of view, Celeste of course activates her problem tackling competency as well as aspects of her reasoning competency. But above all, we are interested in the activation of her mathematical communication competency. She is able to interpret the interlocutors’ mathematical statements and expressions. Also, she is able to express herself and her reasoning, using both words as well as by writing in the table (Fig. 1). Surely, the dialogue happens between senders (us) and receiver (Celeste) with rather different backgrounds and prerequisites. Still, this asymmetry seems not to bother Celeste, rather she seems to profit from it in the sense that she is able to gain the information needed from us in order for her to make sense of the problem and eventually target a solution.

Celeste's solution of a “maths counsellor” detection test task

Another task that we gave to Celeste, is taken from a so-called “detection test” (Jankvist & Niss, 2017) used to detect upper secondary students with mathematics-specific learning difficulties as part of the Danish “maths counsellor” program at Roskilde University (Jankvist & Niss, 2015). Out of 303 upper secondary school mathematics students (1st, 2nd and 3rd year mixed, 16-19 years of age), almost 46% got this wrong for various different reasons (Jankvist & Niss, in review).

Question: Look at the photograph. How high is the building in front? Justify your answer.



Figure 2. The picture accompanying the question above. Taken from Detection test 3 at Roskilde University's "maths counsellor" program (Jankvist & Niss, in review).

To some extent a solution to this task may also rely on proportional reasoning, or scaling at least. Essentially, there are two ways of attacking it: either one may use a person (the one closest to the building) as a yardstick for measuring; or one may estimate the height of a floor and count the number of floors in the building. Celeste rather quickly aims for the first one.

Uffe: Let's try this one [*displays the picture accompanying the question*]

Celeste: Yeah!

Uffe: What does it say? [*points to the text of the question*]

Celeste: It says to look at the picture. [*reads the question aloud*]

Uffe: So, "the building in front" is the blue one there. [*points to the picture*] How high is it? How could we find out?

Celeste: You could look at the people.

Uffe: You could? Okay. What would you do then?

Celeste: Well, maybe find something that is as tall as the person. And then you could... [*grabs a dice from the table, and begins measuring on the picture using this—but then stops again*]

Celeste: Or one could ... like this ...

Here we see that Celeste immediately is "getting in contact" with the problem and paying attention to the picture. From the information gathered from the elements in the picture, she is "locating" the clue of inquiring. She "identifies" the mathematical idea of using a unit to

solve the problem. She partly expresses a knowledge of a person's height and uses hands-on in form of a dice to measure the height of the building. And she begins "thinking aloud". At this point in time, Celeste has essentially "solved" the problem; that is she has devised a model which may potentially lead to a correct solution. What is left for her to do is, firstly, to estimate numbers to put into her model, and, secondly, to justify the model. Based on our experience with Celeste, and when she seems to be stuck, we invite to conversation. And indeed this appears to bring her further in her process of tackling the problem at hand.

Peter: How tall do you think a person is there? Approximately?

Celeste: Maybe 1 meter and ...

Peter: How tall are you?

Celeste: I'm 1 ... I'm usually good at this stuff. I'm 1 ... I think, I'm ... I simply can't remember right now.

Peter: Are you 3 meters tall?

Celeste: No! I'm most certainly 1 meter and something...

Peter: 1 meter and something?

Celeste: 30, I think. Yes, 30 [centimeters] or something like that.

Peter: What if that guy there next to the house is about 2 meters? [...] the guy in the red [sweater]. Let's say he is 2 meters tall, how high is the house then?

Celeste: Then it's...

Peter: You can measure with something, if you want...

Celeste: Well... [*picks up a pencil and measures with the tip of it*] Maybe 22 meters, I think.

Peter: Yes, that's not bad. [...]

After Celeste has made a correct reasoning about a way to solve the problem, a new dialogue takes place where she again is "thinking aloud" and tries to find a reasonable unit for her measuring. We see some "challenging" in the dialogue about the height of a person and her own height, yet she "advocates" by using hands-on the picture with a unit, pencil tip, for an answer. 22 meters is indeed a good answer. Had we, provided a better estimate of the man's height, say, 180 cm, we might have obtained a bit more realistic answer. But that is of less importance in this particular context. The important thing is that Celeste executes her strategy and is provided the opportunity to articulate it along the way. Next, we turn to the final and crucial part of the task, namely for Celeste to justify her answer, i.e. she has to activate her communication competency in order to account for her previous reasoning as part of building a model to tackle the problem.

Uffe: Why is it smart to pick the guy in the red sweater?

Celeste: He is closest to the building, it seems.

Uffe: Why is that important?

Celeste: Well, it could cheat a little...

Uffe: What happens if you pick someone who is not close to the building?

Celeste: Then you don't get it right... It's like if you look from really far away, then it looks like it's really small...

Here Celeste makes an “evaluation” to argue that her answer is indeed reasonable. Although maybe not articulated completely clear-cut, the above statement shows that Celeste has a clear perception of her model being correct and that she also has a sound notion of perspectives in pictures, which is evident from her statement that the perspective might “cheat a little”.

Teachers and gifted students—a matter of communication

As in the case of Celeste, curiosity is often the predominant characteristic of mathematically gifted students. This recurs in both interviews with parents and teachers as well as the majority of researchers who describe characteristics of mathematically giftedness (Pettersson, 2011). But it is not this curiosity which is in focus when teachers are asked to identify such students. What usually triggers a teacher’s attention as an indicator of mathematically giftedness is the ability to work very fast, finish before one’s classmates, provide only correct answers to the questions posed, work independently, and more or less effortlessly do well on tests. What Pettersson (2011) and others point to is that the mathematically gifted students might very well share these characteristics, but that they far from make up a homogeneous group. One of the characteristics that make them into an inhomogeneous group is their very different capabilities when it comes to expressing and communicating their own mathematical thinking. Not least in early primary school there are students who can come up with the correct result without being able to explain their line of thought in arriving at it. There are also students who can come up with algorithms, or even use tools and aids in obtaining their answers, but generally their arguments are incoherent. Only very few of these young students are able to formulate and generalize their reasoning in the form of a step-by-step account of their solution procedure and the ideas and associations they have encountered during this. By putting more focus on the student’s communication competency as part of the solution procedure, a teacher may gain an insight into the individually gifted student’s thinking and in that way obtain an instrument to identify this student’s special need for support as a student in the inhomogeneous group of mathematically gifted students.

We should like to point to two observations. Firstly, in several studies, mathematics teachers express their own lack of competences to communicate with mathematically gifted students and that they often feel helpless in their attempts to do so. In her study, Pettersson (2011) found that:

There are even teachers who are nervous that they do not suffice for the students who find the subject particularly easy, who do not have sufficient competence, and who do not receive support from the school management. (p. 6, our translation from Swedish).

Also from the TMTM project, we know that it is difficult for many teachers to facilitate dialogues with the students, and especially when it comes to mathematically gifted students. When we think about the classical version of the didactical triangle and the asymmetric relationship between teacher and student, it is the teacher who possesses the knowledge and who is in charge of steering the dialogue (e.g. Rezat & Strässer, 2012). But when the dialogue is between a teacher and a mathematically gifted student, then the opposite

situation is not unusual. Although such students may be few in number in early primary mathematics education, they do exist as we have seen from the example with Celeste. Secondly, when going through the literature we have not been able to find any studies specifically focusing on the communication between mathematically gifted students and teachers—only between gifted children and their parents. Taken together, these two observations suggest that it makes up a real and critical problem for many mathematics teachers to teach mathematically gifted students, because the teachers are not able to communicate with such students in ways that support their learning.

In our dialogues with Celeste, we tried to follow the Socratic method as well as the suggestions of Pólya and the framework of Skovsmose and Alrø as background for structuring our conversations with her. Our point is that the teachers' abilities to implement dialogues—and analyze these—are vital for the gifted students' development in mathematics. Teachers need to be able to see their students' utterances in the dialogues as indicators of potentials for learning mathematics. Our claim is that *communication* is an indicator which there should be paid more attention to in the descriptions of gifted students. Not least in the way that Skovsmose and Alrø (2002) talk about mathematical dialogue and learning, we find theoretical perspectives that can come to act as guidelines for the way in which teachers could be educated to become *competent* to engage in dialogues with mathematically gifted students. Such effort, we believe, would also enable teachers to recognize other types of students as mathematically gifted than those who merely answer quickly and correct. It could enable them to “see” mathematically gifted students such as Celeste, who are able to reason deeply and communicate about their mathematical problem tackling processes and results.

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